

- [1] Consider the following model for the growth of a city: The shape of the city always remains roughly circular so that the maximum travel time between two locations in the city is proportional to the diameter of the city. The population of the city is proportional to the area of the city. The rate of increase of the city's population is inversely proportional to the maximum travel time.

(i) This model predicts that the population of the city $P(t)$ satisfies the differential equation

$$\frac{dP}{dt} = \frac{K}{\sqrt{P(t)}}$$

where K is constant. Explain why.

(ii) Verify (by differentiating) that $P(t) = (\frac{3}{2}Kt + C)^{2/3}$ is a solution of the differential equation in part (i).

(iii) If the population of the city was 5000 in 1900 and 20,000 in 1959, what is the predicted population of the city in the year 2000?

- [2] Assume that influenza spreads through the university community at a rate proportional to the product of the number of those infected and the number of those not yet infected. If the total number of students at Carleton is 22000 and $P(t)$ is the number of students infected after t days, express the preceding statement as a differential equation.

- [3] A certain classroom has volume 170 cubic meters. When the concentration of carbon dioxide (CO_2) in the air reaches 0.16%, the ventilation system is activated. This system brings in air containing 0.04% CO_2 at a rate of 28 cubic meters per minute and drives out the mixture of fresh and stale air at the same rate. Assuming complete and instantaneous mixing, what is the percentage of CO_2 in the air in the room after 5 minutes of ventilation?

- [4] A particular man eats a diet of 2500 cal/day; 1200 of them go to basal metabolism (i.e. get used up automatically). He spends approximately 16 cal/kg times his body weight in weight-proportional exercise each day.

(i) Determine how the man's weight varies with time.

(ii) Does the man reach an equilibrium weight? Explain.

[Assume that the storage of calories as fat is 100% efficient and that 1kg fat contains 10^4 calories.]