Elementary Calculus

SUMMARY of INTEGRATION

• Definition of the definite integral: calculates the total change in a function from its rate of change.

• Numerical approximation of definite integrals: average or trapezoidal method.

• Definition of indefinite integral and the Fundamental Theorem of Calculus that says that definite integrals can be evaluated by finding antiderivatives.

• Techniques for finding antiderivatives (also known as indefinite integrals).

(i) Memorize a small table of standard integrals to save time.

(ii) Substitution: eg. $\int \sin(x^2)\, 2x\, dx$; let $u = x^2, du = 2x\, dx$, etc.

(iii) Integration by parts: $\int u\ dv = uv - \int v\ du$. eg. $\int x \sin x\, dx$; let $u = x$, $dv = \sin x\, dx$ so $du = dx$ and $v = -\cos x$ etc.

(iv) Trigonometric integrals: using trig. identities to simplify integrals like

$$\int \sin^4 x\, dx \text{ and } \int \sec^3 x \tan x\, dx.$$

(v) Trigonometric substitutions: subsitutions like $x = a \sin u$ or $x = a \tan u$ to change a square root of a quadratic into a trig integral. eg. $\int \sqrt{a^2 - x^2}\, dx$; let $x = a \sin u$. eg. $\int \sqrt{x^2 + 9}\, dx$; let $x = 3 \tan u$.

(vi) Partial fractions:used on $\int \frac{\text{polynomial}}{\text{polynomial}}$. For example, write

$$\frac{x^2 - 3x + 1}{(x-2)(x^2 + 6x + 13)} = \frac{A}{x-2} + \frac{Bx + C}{x^2 + 6x + 13}$$

Solve for A, B, C, complete the square and integrate to get a ln term and an inverse tan term.

• Applications: Area and volume calculations; solving simple differential equations.

• Skills & tricks: – completing the square.

– handling the limits during substitution in a definite integral.

– (Add your own)

–