

Some Review Problems

[1] Calculate

(a) $\int \frac{x^2+5x+2}{(x+1)(x^2+1)} dx$ (b) $\int \frac{x^5+2}{x^2-1} dx$ (c) $\int \frac{1}{x^2+10x+29} dx$ (d) $\int \frac{1}{x^2+10x+21} dx$

[2] Calculate $\frac{dy}{dx}$

(a) $y = (\sin x)^{x+5}$ (b) $x^2y = x^3 + y \ln x$ (c) $y = [\ln(1+x^3)]^{\sqrt{x}}$

[3] (a) Calculate the integral $\int_3^7 \ln x dx$ by finding an antiderivative.

(b) Estimate the same integral using the “average” or trapezoidal method with 4 subintervals. (Your two answers should be close!)

[4]

(a) $\int \frac{t^6-t^2}{t^4} dt$ (b) $\int \cos^2(5x) dx$ (c) $\int \frac{1}{\sqrt{16-6x-x^2}} dx$ (d) $\int \frac{2x+5}{\sqrt{16-6x-x^2}} dx$
 (e) $\int \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$ (f) $\int x^2 \ln(x) dx$ (g) $\int \frac{x+1}{x^2+1} dx$ (h) $\int_1^3 \frac{\cos(\ln(x))}{x} dx$

[5] Iron-55 has a half life of 2.7 years. How long will it take for a 100 gm sample of radioactive iron to decay to 20 gm ?

[6] More fast breeding bacteria! Suppose we are culturing a population of bacteria with great skill so that the number of cells grows exponentially. After 2 hours there are 6×10^4 and after 8 hours there are 15×10^4 bacteria. How many were present originally? At what time had the population exactly doubled in number?

[7] (a) $\lim_{x \rightarrow \infty} \frac{x^2}{\ln x}$ (b) $\lim_{x \rightarrow 0} \frac{3x - \sin x}{x}$ (c) $\lim_{x \rightarrow \infty} \frac{x - \ln x}{x^2}$ (d) $\lim_{x \rightarrow \pi} \frac{1 + \cos x}{1 + \sin x}$

[8] Let $f(x) = \frac{x+1}{x^2} e^x$.

- (a) Find the intervals where f is positive and where it is negative.
- (b) Show that the derivative is $f'(x) = \frac{x^2-2}{x^3} e^x$.
- (c) Find the intervals where f is increasing and decreasing.
- (d) Find and classify all the critical points. Provide the details and justifications.
- (e) Find the limits as x goes to $\pm\infty$.
- (f) Identify any vertical asymptotes.
- (g) **Sketch** the graph of $f(x)$.